Dealing with uncertainty in experimental data

Recall that if measurements are accurate, the average of the measurements will get closer to the actual value.

Example: We measure the time it takes water to boil 5 times

Trial #1	86.34 seconds
Trial #2	89.34 seconds
Trial #3	82.56 seconds
Trial #4	90.23 seconds
Trial #5	85.26 seconds

What is the average time it took to boil the water?

$$\frac{86.34 + 89.34 + 82.56 + 90.23 + 85.26}{5}$$

$$= 86.746 \approx 86.75 \text{ sec}$$

In reporting our uncertainty, we need to balance two things

- A large range of values for measurements makes us uncertain about the average.
- A large number of measurements makes us more certain about the average.

A formula for uncertainty in the average is

$$\frac{\text{max value} - \text{min value}}{2\sqrt{\#\text{ of measurements}}}$$

Note that more measurements means less uncertainty and more range between max and min values means more uncertainty.

We round uncertainty to 1 sig fig, and round the average to the place value of the uncertainty

Example: We measure the distance a ball rolls in 6.000 seconds, 10 times.

Trial #1	63.1 cm
Trial #2	63.2 cm
Trial #3	63.2 cm
Trial #4	62.9 cm
Trial #5	63.2 cm
Trial #6	63.4 cm (largest)
Trial #7	63.1 cm
Trial #8	62.9 cm
Trial #9	62.8 cm (smallest)
Trial #10	63.4 cm

What is the average distance the ball rolled with uncertainty reported?

(1) Determine average

$$\frac{63.1 + 63.2 + 63.2 + 62.9 + 63.2 + 63.4 + 63.1 + 62.9 + 62.8 + 63.4}{10}$$

$$= 63.12\,\text{cm}$$

(2) Determine uncertainty

$$\frac{63.4 - 62.8}{2\sqrt{10}} = 0.094868\ldots$$

$$\approx 0.09\,\text{cm}$$

↖ 1 sig fig

(3) Write average with uncertainty

$$63.12 \pm 0.09\,\text{cm}$$

Practice

1. An experiment is performed to determine the force of an explosion by measuring the depth of the crater left after the explosion. The experiment is performed 3 times with the following results. Record the average with uncertainty.

	Depth
#1	2.34 m
#2	3.52 m
#3	1.95 m

Average: $\frac{2.34m + 3.52m + 1.95m}{3}$

$= 2.603333\ldots m$

Uncertainty $= \frac{3.52 - 1.95}{2\sqrt{3}} = 0.45321 \approx 0.5\,m$

Average with uncertainty $= 2.6\,m \pm 0.5\,m$

2. An experiment is performed to determine if salt water boils faster than unsalted water. 6 samples of each types of water are boiled and the time to boil recorded. Determine the average with uncertainty for the time it takes to boil each and what can be concluded from the data.

Salted	Time (sec)
#1	65.23 sec
#2	67.26 sec
#3	62.96 sec
#4	70.11 sec
#5	65.24 sec
#6	67.23 sec

avg: 66.33833 sec

Uncert: $\frac{70.11 - 62.96}{2\sqrt{6}} = 1.4594 \approx 1$

66 ± 1 sec

Unsalted	Time (sec)
#1	64.52 sec
#2	64.95 sec
#3	64.64 sec
#4	65.12 sec
#5	64.74 sec
#6	67.84 sec

avg: 65.302 sec

Uncertainty: $\frac{67.84 - 64.52}{2\sqrt{6}} = 0.5776 \approx 0.7$ sec

65.3 ± 0.7 sec